Mass matrix parametrisation for Pseudo-Dirac neutrino

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Introduction

Starting with classical mass matrix – first example is from analytical mechanics:

Kinetic energy for mechanical system

$T = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}\dot{\mathbf{q}}$

And mass matrix can be a function of generalised variables $\mathbf{M} = \mathbf{M}(\mathbf{q})$. If the variables are fields, we have:

Lagrangian mass term for two spinors χ and η

$\mathcal{L}_{mass} = \frac{1}{2}\begin{pmatrix}\chi & \eta\end{pmatrix}\mathbf{M}\begin{pmatrix}\chi \\ \eta\end{pmatrix}$

where mass matrix has the form $\mathbf{M} = \begin{pmatrix} A & M \\ M & B\end{pmatrix}$ and M, A, B are 2x2 matrices.

General spinor formalism

For the most general free field case «Weyl-Majorana-Dirac equation» is

$$i\sigma_\mu \partial^\mu \psi_L - \eta_{D,R} m_{D,R} \psi_R - \eta_L m_L (i\sigma_2) \psi_L^* = 0$$

$$i\bar{\sigma}_\mu \partial^\mu \psi_R - \eta_{D,L} m_{D,L} \psi_L - \eta_R m_R (i\sigma_2) \psi_R^* = 0$$

with non-negative mass terms m and phase terms $\eta = e^{i\varphi}$ from unitary group $U(1)$. Defining $\tilde{m} = \eta m$ and $\psi_R = \begin{pmatrix} \psi_1 + i\psi_2 \\ \psi_3 + i\psi_4 \end{pmatrix}$ $\psi_L = \begin{pmatrix} \psi_5 + i\psi_6 \\ \psi_7 + i\psi_8 \end{pmatrix}$ this equation can be transformed into the form [arXiv:1605.00146]:

$$\Box\Phi + \hat{M}^2 \Phi = 0$$

where $\Phi = (\psi_1 .. \psi_8)^T$
For now let us assume the simple case $m_{D,L} = m_{D,R} = m_D$

Spinor mass matrix

Mass matrix is positive semi-definite Hermitian matrix of the form:

General spinor mass matrix

$$\hat{M}^2 = \begin{pmatrix} M_R & 0 & 0 & A \\ 0 & M_R & -A & 0 \\ 0 & -B & M_L & 0 \\ B & 0 & 0 & M_L \end{pmatrix}$$

$$M_R = \begin{pmatrix} \nu_1 + m_R^2 & -\nu_2 \\ \nu_2 & \nu_1 + m_R^2 \end{pmatrix} \; M_L = \begin{pmatrix} \nu_1 + m_L^2 & -\nu_2 \\ \nu_2 & \nu_1 + m_L^2 \end{pmatrix}$$

$$B = \begin{pmatrix} \mu_1 & \mu_2 \\ \mu_2 & -\mu_1 \end{pmatrix} \; A = \begin{pmatrix} k & 0 \\ 0 & -k \end{pmatrix}$$

where $\tilde{m}_D \tilde{m}_L + \tilde{m}_D^* \tilde{m}_R = k \geqslant 0$ and $\tilde{m}_D^* \tilde{m}_L + \tilde{m}_D \tilde{m}_R = \mu_1 + i\mu_2$
$\tilde{m}_D^2 = \nu_1 + i\nu_2$ This matrix has four doubly degenerate eigenvalues. Coonsidering real and positive m_R and m_D and complex m_L we are down to just two eigenvalues.

Introduction

Now consider χ and η the left- and right-handed neutrino fields ν_L and ν_R. We can work with two Majorana neutrinos considering $\nu_R = \nu_L'^C$.

Then $\mathbf{M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$

- in the case of $m_L = m_R$ we have a pair of eigenvalues $m_D \pm m_L$, mixing angle between ν_L and ν_R is given by $tan2\theta = \frac{2m_D}{m_R - m_L} = \frac{\pi}{4}$
- in the case of $m_L = m_R = 0$ we have a pure Dirac neutrino.
- in the case of $m_L, m_R \ll m_D$ we have a Pseudo-Dirac scenario.

In general, neutrino can have Majorana and Dirac parts

$$\mathcal{L}^{D+M}_{mass} = \mathcal{L}^{D}_{mass} + \mathcal{L}^{L}_{mass} + \mathcal{L}^{R}_{mass}$$

and Dirac neutrino can be represented as two Majorana neutrinos. Left-handed neutrinos are concerned active while right-handed are sterile i.e. they are singlets under $SU(2)_L \times U(1)_Y$.

Pseudo-Dirac neutrino

For the Pseudo-Dirac neutrino the symmetry of mass matrix is not the symmetry of the weak interaction. One can construct

Pseudo-Dirac neutrino decomposition

$$\psi_{\pm L} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \eta_1 \pm i\eta_2 \end{pmatrix} = \frac{1}{\sqrt{2}}(N_{1L} \pm iN_{2L}) \rightarrow \frac{1}{\sqrt{2}}(N_{1L} \pm e^{i\varphi} N_{2L})$$

$$\psi_{\pm R} = \frac{1}{\sqrt{2}} \begin{pmatrix} -i\sigma^2(\eta_1^* \pm i\eta_2^*) \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}(N_{1L}^C \pm iN_{2L}^C) \rightarrow \frac{1}{\sqrt{2}}(N_{1L}^C \pm e^{i\varphi} N_{2L}^C)$$

for a pair of almost degenerate mass Majorana neutrino with opposite CP sign and lepton number not being conserved in higher order weak interaction.

Because of the small value of mass matrix distortions the mixing angle between two Majorana neutrinos is $\sim \frac{\pi}{4}$.

Pseudo-Dirac neutrino motivation

Experimental evidences suggesting sterile neutrino with mass $\sim 1eV$ can be explained in the simpliest way in 3+1 neutrino model. However standard unitary 3+1 data fit suffers from strong tension between MINOS and MINOS+ bound on $\overset{(-)}{\nu}_\mu$ disappearance [arXiv:1710.06488] and LSND&MiniBooNE $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ appearance [arXiv:1801.06467, arXiv:1803.10661].

- 3+1 non-unitary mixing scenario [C. Giunti (2019) arXiv:1904.02093] explains short-baseline disappaearance experiments however the anomalies observed in LSND and MiniBooNE [hep-ex/0104049, arXiv:1805.12028] experiments remain unexplained.
- 3+2 scenario can be studied in general framework of 3 active and 3 sterile neutrino.

The matter is under investigation in the ongoing STEREO, PROSPECT, SoLid and Neutrino-4 experiments.

Pseudo-dirac oscillations

To illustrate potentially observable differences between Dirac and Pseudo-Dirac scenario we will simulate oscillations for T2K experiment parameters:

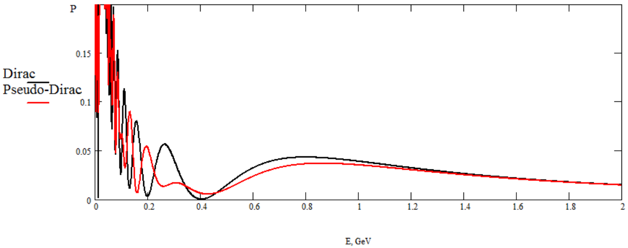
- $L = 295km$ and $E \leq 2GeV$.
- $\delta = -\frac{\pi}{2}$ and $sin^2\theta_{12} = 0.307$ $sin^2\theta_{23} = 0.5$ $sin^2\theta_{13} = 0.218$.
- $\Delta m_{12}^2 = 7.53 \cdot 10^{-5} eV^2$ $\Delta m_{23}^2 = 2.44 \cdot 10^{-3} eV^2$.
- normal mass hierarchy.

Please note that neutrino beam in T2K experiment has energy distribution with maximum at $0.6GeV$ and almost all neutrinos have energy in the interval $0.5 \div 1GeV$.
Using the results from [M. Kobayashi, C.S. Lim (2000) hep-ph/0012266] it is easy to draw $\nu_\mu \to \nu_e$ oscillation probability.

Pseudo-dirac oscillations

Assume that mass eigenvalues splitting for Pseudo-Dirac neutrino is given by $m^2_{iS,A} = m^2_i \pm \epsilon_i m_i$ where $\epsilon_i = 0.1$ Then oscillatin probability is

$$P(\nu_\alpha \to \nu_\beta) = \frac{1}{4}\left|\sum_{j=1}^{3} U_{\beta j}(e^{i\frac{m^2_{jS}}{2E}t} + e^{i\frac{m^2_{jA}}{2E}t})\,U^*_{\alpha j}\right|^2$$



Please also note that energy spectrum now depends on the absolute mass of neutrino because of the splitting!

Mass matrix parametrisation

$M^\dagger M$ diagonalization is the case for chirality preserving processes. In general, 6x6 mass matrix diagonalization gives 15 mixing angles, multiple violating CP phases and 6 eigenvalues. Under Pseudo-Dirac assumption this can be approximated by ordinary 3x3 PMNS matrix.

$$M^\dagger M \simeq \begin{pmatrix} m_D^\dagger m_D & m_L^* m_D^T + m_D^\dagger m_R^* \\ m_D^* m_L + m_R m_D & m_D^* m_D^T \end{pmatrix}$$

consider bi-unitary transformation $U_R^\dagger m_D U_L = diag(m_1, m_2, m_3) = m$ then $V = \begin{pmatrix} U_L & 0 \\ 0 & U_R^* \end{pmatrix}$ and

$$V^\dagger (M^\dagger M) V = \begin{pmatrix} m^2 & U_L^\dagger m_L^\dagger U_L^* m + m U_R^\dagger m_R^* U_R^* \\ m U_L^T m_L U_L + U_R^T m_R U_R m & m^2 \end{pmatrix}$$

Mass matrix parametrisation

If we completely ignore off-diagonal parts then it is just Dirac scenario with doubly-degenerate eigenvalues. In the first order approximation assume that each pair takes the form $\begin{pmatrix} m_i^2 & \epsilon_i^* m_i \\ \epsilon_i m_i & m_i^2 \end{pmatrix}$

Now we obtain 6 mass eigenstates

$$\nu_{iS} = \frac{1}{\sqrt{2}}(\nu_{iL} + e^{i\varphi_i}\nu_{iR}) \; \nu_{iA} = \frac{1}{i\sqrt{2}}(\nu_{iL} - e^{i\varphi_i}\nu_{iR})$$

such that $e^{i\varphi_i} = \frac{\epsilon_i}{|\epsilon_i|}$ for decomposition on slide 6 and mass eigenvalues given by $m_{iS,A}^2 = m_i^2 \pm \epsilon_i m_i$.

Another approach

Another method for diagonalization M itself is comepletely removing left-handed Majorana spinor inside the Dirac one – mass matrix takes the form $M = \begin{pmatrix} \mathbf{0} & m_D^{'} \\ m_D^{'} & M_s \end{pmatrix}$ In [arXiv:0906.1611] it is shown that the appropriate diagonalizing transformation is given in form

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} U^{\dagger} & 1 \\ U & 1 \end{pmatrix} \begin{pmatrix} 1 & \delta \\ -\delta^{\dagger} & 1 \end{pmatrix}$$

where U diagonalises $m_D^{'}$ and $\delta = U(\epsilon/2 + \varepsilon)$, $\varepsilon^T = -\varepsilon$ and $M_s = 2\epsilon m_D - \varepsilon m_D + m_D \varepsilon$. This produces $M = V^{\dagger} m V$ where $m = \begin{pmatrix} m_D(1+\epsilon) & 0 \\ 0 & -m_D(1-\epsilon) \end{pmatrix}$ Now m^2 in the leading order gives the same eigenvalues as in the previous case: $m_i^2 \pm \epsilon_i^{'} m_i$

Experimental check

Consider the following experiments that can provide more experimental data on neutrino oscillations:

- long-baseline experiments.
- short-baseline experiments.
- precise β-decay and K-capture measurements.
- $\beta\beta$ and $0\nu\beta\beta$ observations.
- atmospheric, solar, galactic and extra-galactic neutrino experiments.

It is also important to consider theoretical models for processes in early universe – the constraints are generally less strict than ones from direct observations but still present.

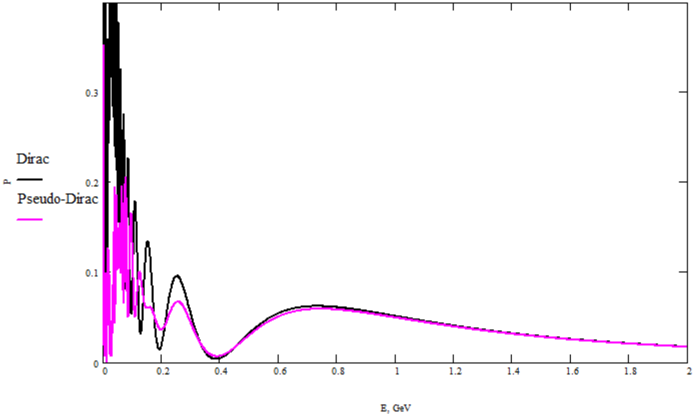
Summary

- The derivation of Pseudo-Dirac neuttrino from general scenario was presented.
- The energy spectrum of neutrino for PD case was simulated.
- It was shown that in the leading order approximation PD neutrino can be effectively described by three ϵ parameters of mass splitting – it is valid for M^2 and M diagonalization.

One then can ask following questions:

- Is it suffice to consider Pseudo-Dirac neutrino to fit observations or general framework is needed?
- What about the mass generation mecanism? Yukawa coupling or multiple scalar fields (like in Zee model) or something else?..

Let us illustrate the difference in energy spectrum for more realistic ϵ_i parameters:



Here we have taken $m_1 = 0.01eV$, $\epsilon_1 = 2.6 \cdot 10^{-3}$,$\epsilon_2 = 4.0 \cdot 10^{-3}$ and $\epsilon_3 = 5.0 \cdot 10^{-3}$.